

Decentralized Dispatch with Distributionally Robust Joint Chance Constraints for Integrated Electrical and Heating System via Dynamic Boundary Response

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Abstract—With the widespread application of combined heat and power (CHP) units, the economic dispatch of integrated electric and district heating systems (IEHSs) has drawn increasing attention. Because the electric power system (EPS) and district heating system (DHS) are generally managed separately, the decentralized dispatch pattern is preferable for the IEHS dispatch problem. However, many common decentralized methods suffer from the drawbacks of slow and local convergence. Moreover, the uncertainties of renewable generation cannot be ignored in a decentralized pattern. Additionally, the most commonly used individual chance constraints in distributionally robust optimization cannot consider safety constraints simultaneously, so the safe operation of an IEHS cannot be guaranteed. Thus, distributionally robust joint chance constraints and robust constraints are jointly introduced into the IEHS dispatch problem in this paper to obtain a stronger safety guarantee, and a method combined with Bonferroni and conditional value at risk (CVaR) approximation is presented to transform the original model into a quadratic program. Additionally, a dynamic boundary response (DBR)-based distributed algorithm based on multiparametric programming is proposed for a fast solution. Case studies showcase the necessity of using mixed distributionally robust joint chance constraints and robust constraints, as well as the effectiveness of the DBR algorithm.

Index Terms—Decentralized optimization, distributionally robust optimization, integrated electric and district heating systems, joint chance constraint, multiparametric programming.

NOMENCLATURE

A. Indices and Sets

$\mathcal{E}^{\text{WD}}$	Index set of wind farms in EPS.
$\mathcal{E}^{\text{TU}}$	Index set of thermal units in EPS.

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$\mathcal{E}^{\text{LD}}$	Index set of loads in EPS.
$\mathcal{E}^{\text{nd}}$	Index set of nodes in EPS.
$\mathcal{E}^{\text{line}}$	Index set of lines in EPS.
$\mathcal{H}^{\text{CHP}}$	Index set of CHP units in the DHS.
$\mathcal{H}$	Index set of DHS areas.
$\mathcal{T}$	Index set of dispatch periods.

B. Parameters

$\hat{p}_{j,t}^{\text{WD}}$	Forecasted active power of wind farm j in period t .
$c_{j,t}^{\text{WD}}$	Penalty coefficient of the curtailment penalty of wind farm j in period t .
$D_{k,t}$	Demand of electric load k in period t .
$\bar{u}_i^{\text{TU}}/\bar{d}_i^{\text{TU}}$	Maximum upward/downward spinning reserve capacity adjustment of thermal unit i .
$U_i^{\text{TU}}/D_i^{\text{TU}}$	Upward/downward ramping capacity of thermal unit i in a single period.
$\bar{P}_i^{\text{TU}}/\underline{P}_i^{\text{TU}}$	Maximum/minimum power of thermal unit i .
$\bar{P}_{j,t}^{\text{WD}}/\underline{P}_{j,t}^{\text{WD}}$	Maximum/minimum power of wind farm j in period t .
$\bar{L}_l$	Line flow limit of transmission line l .
$M_{jl}/M_{il}/M_{gl}$	Generation shift distribution factors of wind farms/thermal units/CHP units.
M_{kl}	Generation shift distribution factors of loads.
$c_{i1}^{\text{TU}}/c_{i2}^{\text{TU}}/c_{i3}^{\text{TU}}/c_i^r$	Cost coefficients of thermal unit i in EPS.
$\varepsilon_1/\varepsilon_2$	Risk parameters.
ρ	Wasserstein radius.
$\mathbb{E}_{\mathcal{P}}(\cdot)$	Expectation operator.
$\mathbb{I}_{\mathcal{X}}(\cdot)$	Indicator function of set $\mathcal{X}$. If $x \in \mathcal{X}$, $\mathbb{I}_{\mathcal{X}}(\cdot) = 0$; otherwise, $\mathbb{I}_{\mathcal{X}}(\cdot) = 1$.
$f_{\text{E}}(\cdot)$	Objective function of EPS.
$f_{\text{H}}^h(\cdot)$	Objective function of DHS h .

C. Random Variables

$\xi_{j,t}^{\text{WD}}$	Forecast error of wind farm j in period t .
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D. Decision Variables

$p_{i,t}^{\text{CHP}}/h_{i,t}^{\text{CHP}}$	Electricity/heating production of CHP unit i in period t .
$\alpha_{i,t}^k$	Coefficient of the k th extreme point of CHP unit i in period t .
$\hat{p}_{i,t}^{\text{TU}}$	Reference base points of the active power of thermal unit i without forecast error in period t .
$p_{i,t}^{\text{TU}}$	Active power of thermal unit i under the realization of renewable energy in period t .
$\gamma_{i,t}$	Participation factors of thermal unit i in period t .
$d_{i,t}^{\text{TU}}/u_{i,t}^{\text{TU}}$	Upward/downward spinning reserve capacity of thermal unit i in period t .
$w_{j,t}^{\text{WD}}$	Curtailment penalty of wind farm j in period t .

Notation: $\|\cdot\|_1$ denotes the 1-norm of the vector; $\|\cdot\|_\infty$ denotes the ∞ -norm of the vector. Other notations are defined as needed.

I. INTRODUCTION

THE energy crisis and environmental problems have become increasingly prominent in recent years, so clean and efficient energy utilization has drawn extensive attention [1]–[3]. Combined heat and power (CHP) units can generate heating and electricity simultaneously, opening new possibilities for improvements in energy efficiency [4]. CHP units have been widely deployed worldwide. For example, in the US, CHP units are supposed to account for 20% of the total installed generating capacity by 2030 [5]. With the widespread application of CHP units, the electric power system (EPS) is closely combined with the district heating system (DHS), and integrated electrical and heating systems (IEHSs) have drawn increasing attention.

Economic dispatch is important for IEHS to optimally utilize its generation resources and minimize operating costs. Some researchers have used mathematical programming algorithms [6] or intelligent algorithms [7] to solve IEHS dispatch problem centrally. However, the EPS and DHS are generally managed separately in the real world. In this situation, it is generally difficult for the system operators to exchange detailed internal topology or operation status information with each other, so a decentralized dispatch pattern may be preferable for IEHS dispatch problem. Distributed algorithms that have been tested for IEHS dispatch problems can be roughly divided into two types: 1) *augmented Lagrangian-based algorithms*, such as the alternating direction method of multipliers (ADMM) algorithm and its variations [8], [9]; and 2) *Karush–Kuhn–Tucker (KKT) condition-based algorithms*, such as heterogeneous decomposition (HGD) [10].

However, many of the above methods suffer from the drawbacks of slow and local convergence. For example, the ADMM algorithm is sensitive to penalty parameters, and inappropriate parameters may lead to substantial computation cost; HGD converges only locally and may fail occasionally. To address these issues, multiparametric programming has been leveraged to design decentralized algorithms for multi-system economic

dispatch problems, including a series of noniterative [11], [12] and iterative versions [13]–[15]. However, the noniterative versions face the “curse of dimensionality” for large systems, while the iterative versions may be computationally inefficient due to the need to take the inverse of a large-scale matrix in each iteration.

Meanwhile, uncertainties of renewable generation were not considered in most of the above decentralized studies [16]. In recent years, the penetration of renewable generation has experienced an upward trend. For example, in Denmark, the penetration of renewable generation has increased from 44% of the total available capacity in 2015 to 55% in 2020 [17]. With the increasing penetration of renewable generation uncertainties, the impact on the entire IEHS dispatch process cannot be ignored [18]. While stochastic optimization [19] and robust optimization [20] have been tested in the literature, the former depends on having accurate decentralized information on uncertainty, and the latter may yield an overly conservative solution. Distributionally robust optimization (DRO), which can reportedly mitigate the above issues, was adopted in [21], [22].

The ambiguity set in DRO can be divided into metric-based and moment-based ambiguity sets. The moment-based ambiguity set is constructed by assuming the mean and covariances of uncertain parameters and is popular due to its better tractability features [23]. However, the assumption of moment-based ambiguity sets may ignore other information in the sample data [24]. Compared with the moment-based ambiguity set, the metric-based ambiguity set does not assume any specific probability distribution of the uncertainties. In metric-based ambiguity sets, the Wasserstein metric-based ambiguity set has many conceptual advantages and tractable reformulations [25], which have recently gained much attention. The Wasserstein metric-based DRO has been adopted in the planning [26] and dispatch [27], [28] of power systems, and some researchers have adopted it in integrated energy systems [29]–[31].

As a key component of DRO, chance constraints that probabilistically constrain the safety of the system of interest should be properly formulated. In general, there are two types of chance constraints: *individual* and *joint*. Compared with *individual* ones, *joint* chance constraints consider several safety constraints simultaneously, which can provide stronger security guarantees for the entire system [32]. However, joint chance constraints are generally much more challenging to handle than individual chance constraints. For example, [33] showed that individual chance constraints with only right-hand-side disturbances can be transformed into a convex program, while the corresponding result is available for joint chance constraints only in log-concave distributions. Very recently, some researchers have adopted the joint chance constraint in multi-energy source microgrids [34] and power systems [35], but it has been challenging to use in a decentralized pattern. To the best of our knowledge, applying distributionally robust joint chance constraints (DRJCCs) to an IEHS in a decentralized pattern remains a challenging task.

To address the challenges mentioned above, we propose a decentralized data-driven DRJCC method for the multiperiod IEHS dispatch problem. The contributions are as follows:

1) Distributionally robust joint chance constraints and robust constraints are jointly introduced into the IEHS dispatch problem to obtain a stronger safety guarantee. Specifically, robust constraints are adopted to guarantee some sorts of physical constraints, such as ramping constraints, have to be obeyed 100% against uncertainty, which accurately reflects the practical requirements. To resolve the mixed model of distributionally robust chance constraints and conventional robust constraints, inspired by [36], we present a method combined with Bonferroni and conditional value at risk (CVaR) approximation to transform the original model into a tractable quadratic program model.

2) An efficient dynamic boundary response (DBR)-based decentralized algorithm is proposed to leverage the coupling characteristics between the EPS and DHS. The main idea of the DBR algorithm is DHS operator dynamically responds to the boundary information from the EPS operator by projecting the objective function and constraints of the DHS into the boundary variable space. Meanwhile, a preprocessing step is introduced into the BDR algorithm to improve computational efficiency.

The remainder of this article is organized as follows. A data-driven distributionally robust joint constraint-based IEHS model is formulated in Section II. The tractable reformulation is performed in Section III. Section IV presents the DBR algorithm. Numerical results are discussed in Section V, and conclusions are provided in Section VI.

II. DRJCC MULTIPERIOD DISPATCH MODEL FOR AN IEHS

A. Coupling Structure of the IEHS

The coupling structure of an IEHS is shown in Fig. 1. The IEHS consists of an EPS and several DHSs, which are coupled by CHP units. EPS and DHSs are generally managed by an EPS operator and DHS operators, respectively. The boundary variable between EPS and DHSs is the electricity production of the CHP units. In the following, $\mathbf{x}_E$ denotes the vector of all interval variables in the EPS, including $\hat{p}_{i,t}^{\text{TU}}$, $p_{i,t}^{\text{TU}}$, $\gamma_{i,t}$, $w_{j,t}^{\text{WD}}$, $d_{i,t}^{\text{TU}}$, and $u_{i,t}^{\text{TU}}$; $\mathbf{x}_H^h$ denotes the vector of all decision variables of DHS h , including the heating production of CHP units, the heating production and fuel input of heating boilers, and the

temperature of pipelines and nodes, etc.; and $\mathbf{x}_B^h$ denotes the vector of boundary variables between DHS h and the EPS, including $p_{i,t}^{\text{CHP}}$.

In this paper, the following assumptions are adopted:

1) The potential mismatch due to wind power uncertainty will be mitigated by controlling the thermal units' power in the EPS, which means uncertainty on EPS side will not propagate into the DHS.

2) The uncertainty caused by the loads is not considered because the accuracy of the load forecast is relatively high [37].

B. DRJCC Dispatch Model of the EPS

An EPS consists of traditional thermal units, wind farms, transmission lines, and electrical loads. The uncertainty in the EPS is caused by wind power.

1) Support Polyhedron Set and Ambiguity Set of Uncertainty

In general, the true wind distribution $\mathbb{P}$ is difficult to know, but the decision maker can have the discrete empirical distribution $\hat{\mathbb{P}}_N^t$ according to many historical samples $\xi_{n,t}$, $n = 1, \dots, N$ of wind. Hence, the Wasserstein distance is adopted to construct the ambiguity set based on the empirical distribution. Because the type-1 Wasserstein distance can lead to linear programming reformulation, we choose the type-1 Wasserstein distance to measure the distance between two distributions in this paper. Type-1 Wasserstein distance between two distributions $\mathbb{P}_1$ and $\mathbb{P}_2$ is defined by:

$$\mathbb{W}(\mathbb{P}_1, \mathbb{P}_2) = \min_{\Pi} \left\{ \int_{\mathbb{R}^W \times \mathbb{R}^W} \|\xi_1 - \xi_2\|_1 \Pi(d\xi_1 - d\xi_2) \right\} \quad (1)$$

where ξ_1 and ξ_2 denote the random vectors whose distributions are $\mathbb{P}_1$ and $\mathbb{P}_2$, respectively; Π denotes the joint distribution of ξ_1 and ξ_2 on $\mathbb{R}^W \times \mathbb{R}^W$.

The support polyhedron set of wind in period t is denoted by:

$$\Xi^t = \{\xi_t : H_t \xi_t \leq h_t\}, \quad H_t = [I \quad -I]^T, \quad h_t = [(\hat{p}_t^{\text{WD}} - \hat{p}_t^{\text{WD}})^T (\hat{p}_t^{\text{WD}} - \underline{p}_t^{\text{WD}})^T]^T \quad (2)$$

where we use the notation $\xi_t / \hat{p}_t^{\text{WD}} / \bar{p}_t^{\text{WD}} / \underline{p}_t^{\text{WD}}$ to stack $\xi_{j,t}^{\text{WD}} / \hat{p}_{j,t}^{\text{WD}} / \bar{p}_{j,t}^{\text{WD}} / \underline{p}_{j,t}^{\text{WD}}$ over $j \in \mathcal{E}^{\text{WD}}$; I denotes the identity matrix.

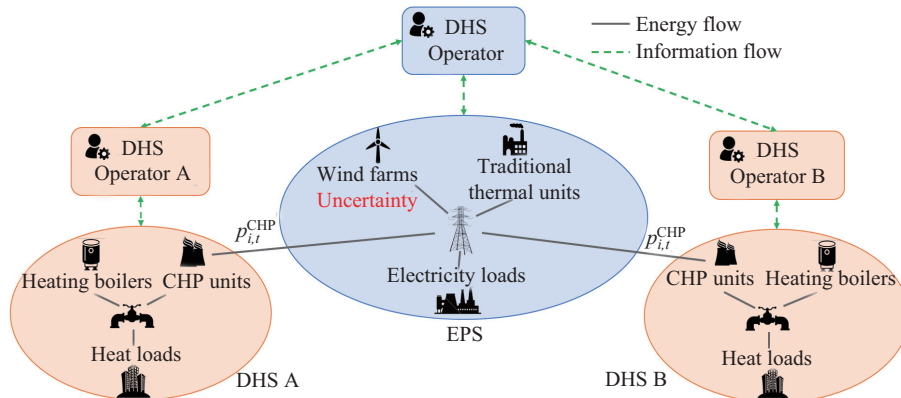


Fig. 1. Structure of an IEHS.

Denote by $\mathcal{M}(\Xi)$ the set of all distributions of wind on the support polyhedron set Ξ . According to (1), the ambiguity set in period t is defined by:

$$\mathcal{P}^t = \{\mathbb{P} \in \mathcal{M}(\Xi^t) : W(\mathbb{P}, \hat{\mathbb{P}}_N^t) \leq \rho\} \quad (3)$$

2) Affine Policy-Based Adjustments of Thermal Units

The renewable energy forecast error can destroy the power balance of the power system. To maintain the power balance, this paper employs an affine policy to adjust the thermal units' power outputs in EPS, and the renewable energy forecast error can be fully mitigated by adjusting the thermal units based on the affine policy:

$$p_{i,t}^{\text{TU}} = \hat{p}_{i,t}^{\text{TU}} - \gamma_{i,t} \sum_{j \in \mathcal{E}^{\text{WD}}} (\xi_{j,t}^{\text{WD}}), \quad i \in \mathcal{E}^{\text{TU}}, \quad t \in \mathcal{T} \quad (4)$$

$$\sum_{i \in \mathcal{E}^{\text{TU}}} \gamma_{i,t} = 1, \quad 0 \leq \gamma_{i,t} \leq 1, \quad i \in \mathcal{E}^{\text{TU}}, \quad t \in \mathcal{T} \quad (5)$$

where $\gamma_{i,t}$ is a decision variable and an optimal linear decision rule is applied for thermal units in the EPS by optimizing $\gamma_{i,t}$.

3) Formulation of EPS Dispatch Model

The dispatch model of the EPS based on the linearized power flow formulation is as follows:

Objective Function:

$$\begin{aligned} \min_{\mathbf{x}_E, \mathbf{x}_B^h} f(\mathbf{x}_E) = & \sum_{t \in \mathcal{T}} \left\{ \max_{\mathbb{P} \in \mathcal{P}^t} \mathbb{E}_{\mathbb{P}} \left\{ \sum_{i \in \mathcal{E}^{\text{TU}}} [c_{i1}^{\text{TU}} (p_{i,t}^{\text{TU}})^2 + c_{i2}^{\text{TU}} p_{i,t}^{\text{TU}} \right. \right. \\ & \left. \left. + c_{i3}^{\text{TU}}] + \sum_{i \in \mathcal{E}^{\text{TU}}} c_i^{\text{r}} (u_{i,t}^{\text{TU}} + d_{i,t}^{\text{TU}}) \right\} + \sum_{j \in \mathcal{E}^{\text{WD}}} c_{j,t}^{\text{WD}} w_{j,t}^{\text{WD}} \right\} \end{aligned} \quad (6)$$

Power Balance Constraints:

$$\begin{aligned} & \sum_{i \in \mathcal{E}^{\text{TU}}} \hat{p}_{i,t}^{\text{TU}} + \sum_{j \in \mathcal{E}^{\text{WD}}} (\hat{p}_{j,t}^{\text{WD}} - w_{j,t}^{\text{WD}}) + \sum_{g \in \mathcal{H}^{\text{CHP}}} p_{g,t}^{\text{CHP}} \\ & = \sum_{k \in \mathcal{E}^{\text{LD}}} D_{k,t} \end{aligned} \quad (7)$$

Wind Curtailment Constraints:

$$w_{j,t}^{\text{WD}} \geq 0, \quad j \in \mathcal{E}^{\text{WD}}, \quad t \in \mathcal{T} \quad (8)$$

Spinning Reserve Constraints:

$$\hat{p}_{i,t}^{\text{TU}} + u_{i,t}^{\text{TU}} \leq \bar{P}_i^{\text{TU}}, \quad 0 \leq u_{i,t}^{\text{TU}} \leq \bar{u}_i^{\text{TU}}, \quad i \in \mathcal{E}^{\text{TU}}, \quad t \in \mathcal{T} \quad (9)$$

$$\bar{P}_i^{\text{TU}} \leq \hat{p}_{i,t}^{\text{TU}} - d_{i,t}^{\text{TU}}, \quad 0 \leq d_{i,t}^{\text{TU}} \leq \bar{d}_i^{\text{TU}}, \quad i \in \mathcal{E}^{\text{TU}}, \quad t \in \mathcal{T} \quad (10)$$

Spinning Reserve (Joint Chance Constraints):

$$\begin{aligned} \min_{\mathbb{P} \in \mathcal{P}^t} \mathbb{P} \left[-d_{i,t}^{\text{TU}} \leq -\gamma_{i,t} \sum_{j \in \mathcal{E}^{\text{WD}}} \xi_{j,t}^{\text{WD}} \leq u_{i,t}^{\text{TU}}, \quad i \in \mathcal{E}^{\text{TU}} \right] \\ \geq 1 - \varepsilon_1, \quad t \in \mathcal{T} \end{aligned} \quad (11)$$

Transmission Line Flow (Joint Chance Constraints):

$$\begin{aligned} \min_{\mathbb{P} \in \mathcal{P}^t} \mathbb{P} \left[-\bar{L}_l \leq \sum_{j \in \mathcal{E}^{\text{WD}}} M_{jl} (\hat{p}_{j,t}^{\text{WD}} + \xi_{j,t}^{\text{WD}} - w_{j,t}^{\text{WD}}) \right. \\ \left. + \sum_{i \in \mathcal{E}^{\text{TU}}} M_{il} \hat{p}_{i,t}^{\text{TU}} + \sum_{g \in \mathcal{H}^{\text{CHP}}} M_{gl} p_{g,t}^{\text{CHP}} \right] \\ \geq 1 - \varepsilon_2, \quad t \in \mathcal{T} \end{aligned} \quad (12)$$

Ramping (Robust Constraints):

$$\begin{aligned} -D_i^{\text{TU}} \leq & \left(\hat{p}_{i,t}^{\text{TU}} - \gamma_{i,t} \sum_{j \in \mathcal{E}^{\text{WD}}} \xi_{j,t}^{\text{WD}} \right) \\ & - \left(\hat{p}_{i,t-1}^{\text{TU}} - \gamma_{i,t-1} \sum_{j \in \mathcal{E}^{\text{WD}}} \xi_{j,t-1}^{\text{WD}} \right) \\ & \leq U_i^{\text{TU}}, \quad \forall \xi_{j,t}^{\text{WD}} \in \Xi^t, \quad \forall \xi_{j,t-1}^{\text{WD}} \in \Xi^{t-1}, \quad i \in \mathcal{E}^{\text{TU}}, \quad t \in \mathcal{T} \end{aligned} \quad (13)$$

The minimax objective function aims to minimize the total system cost in the worst case. The first term denotes the cost of the thermal units, and the second term denotes the cost of reserve capacity. Joint chance constraints (11) and (12) guarantee reserve and line flow constraints can be probabilistically satisfied under the worst-case distribution. Unlike constraints (11) and (12), as a physical constraint of the generators, the ramping constraints must be obeyed 100% of the time against uncertainty. If the ramping constraints are violated, it will make the entire dispatch results infeasible. For the operational feature, a robust constraint formula other than the chance constraint should be adopted in the case of an impractical ramping plan for the generators. Thus, we use robust constraints (12) to guarantee ramping constraints are always satisfied over the support polyhedron set.

C. District Heating System Model

A DHS consists of heat sources, district heat networks, and heating loads. The heat energy comes from the CHP units and heating boilers and flows through the heating network to the heat loads.

Similar to previous studies on IEHSs [9], [10], we consider a common DHS model based on the constant mass flow and variable temperature strategy. Since the forecast errors are assumed to be fully mitigated by adjusting the flexible resources in EPS, DHS model is deterministic. To facilitate the subsequent mathematical derivation, the DHS model is given in a compact form below, and the detailed formulation is available in Appendix A.

$$\begin{aligned} \min_{\mathbf{x}_H^h, \mathbf{x}_B^h} f_H(\mathbf{x}_H^h, \mathbf{x}_B^h) = & (\mathbf{x}_H^h)^T \mathbf{Q}_{HH} \mathbf{x}_H^h + (\mathbf{x}_H^h)^T \mathbf{Q}_{HB} \mathbf{x}_B^h \\ & + (\mathbf{x}_B^h)^T \mathbf{Q}_{BB} \mathbf{x}_B^h + \mathbf{D}_H \mathbf{x}_H^h \end{aligned} \quad (14)$$

$$+ \mathbf{D}_{HB} \mathbf{x}_B^h + e_H \quad (15)$$

$$\mathbf{A}_H^h \mathbf{x}_H^h + \mathbf{A}_{BH}^h \mathbf{x}_B^h = \mathbf{a}_H^h \quad (16)$$

$$\mathbf{B}_H^h \mathbf{x}_H^h + \mathbf{B}_{BH}^h \mathbf{x}_B^h \leq \mathbf{b}_H^h \quad (16)$$

where $\mathbf{A}_H^h$, $\mathbf{A}_{BH}^h$, and $\mathbf{a}_H^h$ and $\mathbf{B}_H^h$, $\mathbf{B}_{BH}^h$, and $\mathbf{b}_H^h$ denote constant coefficient matrices in the linear equality constraints and inequality constraints, respectively.

The objective function of DHS aims to minimize the operating cost of CHP units and heating boilers. Equality constraints (15) consist of CHP unit and heating boiler operation constraints, heating source constraints, temperature mixing constraints, heating loss constraints, temperature quasi-dynamic constraints, inlet temperature constraints, and heat load constraints; inequality constraints (16) consist of the CHP

unit and heating boiler output constraints and temperature security constraints.

D. Integral IEHS Dispatch Model (DRJCC Model)

1) Objective Function

The objective function of the multiperiod IEHS dispatch problem is the total operating cost of the EPS and DHS, including (6) and (14):

$$\min_{\mathbf{x}_E, \mathbf{x}_H^h, \mathbf{x}_B^h} \left\{ f_E(\mathbf{x}_E) + \sum_{h \in \mathcal{H}} f_H^h(\mathbf{x}_H^h, \mathbf{x}_B^h) \right\} \quad (17)$$

2) Constraints

The constraints are all the constraints of the EPS and DHS, which include (4)–(5), (7)–(13), and (15)–(16).

Unlike the model in [9], [10], we consider joint chance constraints (11)–(12) to guarantee that constraints can be probabilistically satisfied simultaneously. Moreover, robust constraints (13) make the original model a mixed distributionally robust chance constraint and robust constraint model.

III. TRACTABLE REFORMULATION

The original dispatch model proposed in Section II is intractable due to the expectation terms in the objective function (17), joint chance constraints (11)–(12), and robust constraints (13). In this section, tractable reformulations for the original DRJCC model are proposed.

A. Convex Reformulation of the Objective Function

To address the quadratic terms in the objective function (6), we apply an approximation method, and the first term in (6) can be transformed to:

$$\begin{aligned} & \max_{\mathbb{P} \in \mathcal{P}^t} \left\{ \mathbb{E}_{\mathbb{P}} \sum_{i \in \mathcal{E}^{\text{TU}}} \left[c_{i1}^{\text{TU}} (p_{i,t}^{\text{TU}})^2 + c_{i2}^{\text{TU}} p_{i,t}^{\text{TU}} + c_{i3}^{\text{TU}} \right] \right\} \\ &= \sum_{i \in \mathcal{E}^{\text{TU}}} \left[c_{i1}^{\text{TU}} (\hat{p}_{i,t}^{\text{TU}})^2 + c_{i2}^{\text{TU}} \hat{p}_{i,t}^{\text{TU}} + c_{i3}^{\text{TU}} \right] \\ &+ \max_{\mathbb{P} \in \mathcal{P}^t} \mathbb{E}_{\mathbb{P}} \left\{ \sum_{i \in \mathcal{E}^{\text{TU}}} \left(-c_{i0,t} \gamma_{i,t} \sum_{j \in \mathcal{E}^{\text{WD}}} \xi_{j,t}^{\text{WD}} \right) \right\} \quad (18) \end{aligned}$$

where $c_{i0,t} = 2c_{i1}^{\text{TU}} p_{i,t}^{\text{TU}*} + c_{i2}^{\text{TU}}$; $p_{i,t}^{\text{TU}*}$ denotes the optimal output of the thermal unit in the case without forecast error, namely, $\xi_{j,t} = 0$. Denote by $\ell_t(\xi_t)$ the expectation term in (18):

$$\begin{aligned} & \max_{\mathbb{P} \in \mathcal{P}^t} \mathbb{E}_{\mathbb{P}} \{ \ell_t(\xi_t) \} \\ &= \max_{\mathbb{P} \in \mathcal{P}^t} \mathbb{E}_{\mathbb{P}} \left\{ \sum_{i \in \mathcal{E}^{\text{TU}}} \left(-c_{i0,t} \gamma_{i,t} \sum_{j \in \mathcal{E}^{\text{WD}}} \xi_{j,t}^{\text{WD}} \right) \right\} \\ &= \max_{\mathbb{P} \in \mathcal{P}^t} \mathbb{E}_{\mathbb{P}} \{ -\mathbf{c}_{0,t}^{\text{T}} \mathbf{Y}_t \mathbf{e}^{\text{T}} \xi_t \} \quad (19) \end{aligned}$$

where we use the notation $\mathbf{Y}_t$ and $\mathbf{c}_{0,t}$ to stack $\gamma_{i,t}$ and $c_{i0,t}$ over $i \in \mathcal{E}^{\text{TU}}$ in period t , respectively; $\mathbf{e}$ denotes a vector whose components are all 1. Because $\ell_t(\xi_t)$ is a proper convex function and Ξ^t is convex and closed, as discussed in [23], the worst-case expectation (19) can be transformed into:

$$\min_{\beta^o, \mathbf{s}_n^o, \boldsymbol{\eta}_n^o, \mathbf{Y}_t} \beta^o \rho + \frac{1}{N} \sum_{n=1}^N \mathbf{s}_n^o$$

$$\begin{aligned} \text{s.t. } & [-\ell_t]^* (\boldsymbol{\pi}_n - \boldsymbol{\nu}_n) + \sigma_{\Xi^t}(\boldsymbol{\nu}_n) - \boldsymbol{\pi}_n^{\text{T}} \boldsymbol{\xi}_{n,t} \leq \mathbf{s}_n^o, \quad (20) \\ & \|\boldsymbol{\pi}_n\|_x \leq \beta^o, \quad n = 1, \dots, N \end{aligned}$$

where $[-\ell_t]^* (\boldsymbol{\pi}_n - \boldsymbol{\nu}_n)$ denotes the conjugate of $-\ell_t$ evaluated at $\boldsymbol{\pi}_n - \boldsymbol{\nu}_n$; σ_{Ξ} denotes the support function of Ξ^t ; and $\boldsymbol{\pi}_n, \boldsymbol{\nu}_n, \beta^o$ and $\mathbf{s}_n^o$ denote auxiliary variables in the objective function. In this paper, $[-\ell_t]^* (\boldsymbol{\pi}_n - \boldsymbol{\nu}_n)$ and $\sigma_{\Xi^t}(\boldsymbol{\nu}_n)$ are [23]:

$$\begin{aligned} [-\ell_t]^* (\boldsymbol{\pi}_n - \boldsymbol{\nu}_n) &= \sup_{\xi} \{ (\boldsymbol{\pi}_n - \boldsymbol{\nu}_n)^{\text{T}} \xi - \mathbf{c}_{0,t}^{\text{T}} \mathbf{Y}_t \mathbf{e}^{\text{T}} \xi \} \\ &= \begin{cases} 0 & \text{if } \boldsymbol{\pi}_n - \boldsymbol{\nu}_n = \mathbf{e} \mathbf{Y}_t^{\text{T}} \mathbf{c}_{0,t} \\ \infty & \text{else} \end{cases} \quad (21) \end{aligned}$$

$$\sigma_{\Xi^t}(\boldsymbol{\nu}_n) = \begin{cases} \sup_{\xi} \boldsymbol{\nu}_n^{\text{T}} \xi \\ \text{s.t. } \mathbf{H}_t \xi \leq \mathbf{h}_t \end{cases} = \begin{cases} \inf_{\boldsymbol{\eta}_n^o \geq 0} \boldsymbol{\eta}_n^{\text{oT}} \mathbf{h}_t \\ \text{s.t. } \mathbf{H}_t^{\text{T}} \boldsymbol{\eta}_n^o = \boldsymbol{\nu}_n \end{cases} \quad (22)$$

where $\boldsymbol{\eta}_n^o$ denotes the auxiliary variable. Equation (22) follows from strong duality, and it holds as the uncertainty set is nonempty.

Thus, by substituting (21) and (22) into (20), the worst-case expectation (20) can be transformed into a conic program:

$$\begin{aligned} & \min_{\beta^o, \mathbf{s}_n^o, \boldsymbol{\eta}_n^o, \mathbf{Y}_t} \beta^o \rho + \frac{1}{N} \sum_{n=1}^N \mathbf{s}_n^o \\ \text{s.t. } & -\mathbf{c}_{0,t}^{\text{T}} \mathbf{Y}_t \mathbf{e}^{\text{T}} \boldsymbol{\xi}_{n,t} + \boldsymbol{\eta}_n^{\text{oT}} (\mathbf{h}_t - \mathbf{H}_t \boldsymbol{\xi}_{n,t}) \leq \mathbf{s}_n^o, \\ & n = 1, \dots, N \\ & \|\mathbf{H}_t^{\text{T}} \boldsymbol{\eta}_n^o + \mathbf{e} \mathbf{Y}_t^{\text{T}} \mathbf{c}_{0,t}\|_{\infty} \leq \beta^o, \quad \boldsymbol{\eta}_n^o \geq 0, \quad n = 1, \dots, N \quad (23) \end{aligned}$$

where N denotes the number of samples in the sample set.

B. Convex Reformulation of Joint Chance Constraints

This section addresses joint chance constraints (11)–(12), and a method combined with Bonferroni and CVaR approximation is presented to transform the joint chance constraints into a system of linear constraints.

Because joint chance constraints (11)–(12) are not coupled in different periods, a generic joint chance constraint is defined as:

$$\Phi := \{ (\mathbf{Y}_t, \mathbf{y}_t) : \min_{\mathbb{P} \in \mathcal{P}^t} \mathbb{P}[\mathbf{A}(\mathbf{Y}_t) \boldsymbol{\xi}_t \leq \mathbf{b}(\mathbf{y}_t)] \geq 1 - \varepsilon \} \quad (24)$$

where we use the notation $\mathbf{y}_t$ to stack variables $(\hat{p}_{i,t}^{\text{TU}}, p_{i,t}^{\text{CHP}}, w_{j,t}^{\text{WD}}, u_{i,t}^{\text{TU}}, d_{i,t}^{\text{TU}})$ over i and j in period t ; $\mathbf{A}(\mathbf{Y}_t)$ and $\mathbf{b}(\mathbf{y}_t)$ are affine mappings. We assume a joint chance constraint involves R affine inequalities, and matrix $\mathbf{A}(\mathbf{Y}_t)$ and vector $\mathbf{b}(\mathbf{y}_t)$ can be decomposed as:

$$\begin{aligned} \mathbf{A}(\mathbf{Y}_t) &= [\mathbf{a}_1(\mathbf{Y}_t), \dots, \mathbf{a}_R(\mathbf{Y}_t)], \\ \mathbf{b}(\mathbf{y}_t) &= [\mathbf{b}_1(\mathbf{y}_t), \dots, \mathbf{b}_R(\mathbf{y}_t)] \quad (25) \end{aligned}$$

The joint chance constraints (24) can also be expressed as:

$$\min_{\mathbb{P} \in \mathcal{P}^t} \mathbb{P}[\mathbf{a}_r(\mathbf{Y}_t)^{\text{T}} \boldsymbol{\xi}_t \leq \mathbf{b}_r(\mathbf{y}_t), r = 1, \dots, R] \geq 1 - \varepsilon \quad (26)$$

According to Bonferroni's inequality, we can split the joint chance constraint (26) into a family of R individual chance constraints:

$$\Theta_{\text{B}} := \left\{ (\mathbf{Y}_t, \mathbf{y}_t) : \max_{\mathbb{P} \in \mathcal{P}^t} \mathbb{P}[\mathbf{a}_r(\mathbf{Y}_t)^{\text{T}} \boldsymbol{\xi}_t \leq \mathbf{b}_r(\mathbf{y}_t)] \geq 1 - \varepsilon_r \quad (r = 1, \dots, R) \right\} \quad (27)$$

where ε_r denotes a set of individual violation tolerances with $\varepsilon_r \geq 0$ and $\sum_{r=1}^R \varepsilon_r = \varepsilon$. In this paper, we set $\varepsilon_r = \varepsilon/R$ for all individual chance constraints as recommended in [38].

However, the individual chance constraint (27) is still difficult to calculate, so we approximate (27) by worst-case CVaR constraints:

$$\Theta_{BC} := \left\{ (\mathbf{Y}_t, \mathbf{y}_t) : \max_{\mathbb{P} \in \mathcal{P}^t} \mathbb{P} - \text{CVaR}[a_r(\mathbf{Y}_t)^T \boldsymbol{\xi}_t - b_r(\mathbf{y}_t)] \leq 0 \right\} \quad (28)$$

It was shown that Θ_{BC} is the best convex inner approximation of Θ_B in [38]. According to [36], the approximate set Θ_{BC} can be reformulated as linear constraints:

$$\Theta_{JCC} = \left\{ (\mathbf{Y}_t, \mathbf{y}_t) : \begin{array}{l} \exists \boldsymbol{\alpha} \in \mathbb{R}^R, \boldsymbol{\eta} \in \mathbb{R}^R, \boldsymbol{\beta} \in \mathbb{R}^{N \times R}, \\ \boldsymbol{\eta}_r \rho + \frac{1}{N} \sum_{n=1}^N \boldsymbol{\beta}_{nr} \leq 0, \boldsymbol{\alpha}_r \leq \boldsymbol{\beta}_{nr}, \\ \boldsymbol{\tau}_{n,r} \geq 0, \\ \|\varepsilon_r \mathbf{H}_t^T \boldsymbol{\tau}_{nr} - \mathbf{a}_r(\mathbf{Y}_t)\|_\infty \leq \varepsilon_r \boldsymbol{\eta}_r, \\ \mathbf{a}_r(\mathbf{Y}_t)^T \hat{\boldsymbol{\xi}}_{n,t} - \mathbf{b}_l(\mathbf{y}_t) + (\varepsilon_r - 1) \boldsymbol{\alpha}_r \\ + \varepsilon_r \boldsymbol{\tau}_{nr}^T (\mathbf{h}_t - \mathbf{H}_t \hat{\boldsymbol{\xi}}_{n,t}) \leq \varepsilon_r \boldsymbol{\beta}_{nr}, \\ n = 1, \dots, N, r = 1, \dots, R \end{array} \right\} \quad (29)$$

where $\boldsymbol{\alpha}$, $\boldsymbol{\eta}$, and $\boldsymbol{\beta}$ denote auxiliary variables in the joint chance constraints.

C. Convex Reformulation of Robust Constraints

In addition to the joint chance constraint, the robust constraint (13) is difficult to handle in the proposed model. This subsection aims to transform the robust constraints into a system of linear constraints.

Unlike joint chance constraints, the robust constraint (12) is coupled in different periods, which can be expressed as:

$$\begin{aligned} \mathbf{c}_t^T \mathbf{z}_t + \mathbf{c}_{t-1}^T \mathbf{z}_{t-1} &\leq \mathbf{d}_t + \mathbf{d}_{t-1} \\ t \in \mathcal{T}, \forall \xi_{j,t}^{\text{WD}} \in \Xi^t, \forall \xi_{j,t-1}^{\text{WD}} \in \Xi^{t-1} \\ [\mathbf{c}_t; \mathbf{d}_t] &= [\mathbf{c}_t^0; \mathbf{d}_t^0] + \sum_{j \in \mathcal{E}^{\text{WD}}} \xi_{j,t}^{\text{WD}} [\mathbf{c}_t^j; \mathbf{d}_t^j] \\ [\mathbf{c}_{t-1}; \mathbf{d}_{t-1}] &= [\mathbf{c}_{t-1}^0; \mathbf{d}_{t-1}^0] + \sum_{j \in \mathcal{E}^{\text{WD}}} \xi_{j,t-1}^{\text{WD}} [\mathbf{c}_{t-1}^j; \mathbf{d}_{t-1}^j] \end{aligned} \quad (30)$$

where we use the notation $\mathbf{z}_t$ to stack $(\hat{p}_{i,t}^{\text{TU}}, \gamma_{i,t})$ over $i \in \mathcal{E}^{\text{TU}}$ in period t ; $\mathbf{c}_t^0$ and $\mathbf{d}_t^0$ denote the matrix that is independent of uncertainty in the constraint, and $\mathbf{c}_t^j$ and $\mathbf{d}_t^j$ denote the matrix associated with $\xi_{j,t}^{\text{WD}}$ in the constraint. In this case, (30) is:

$$\begin{aligned} &\sum_{j \in \mathcal{E}^{\text{WD}}} \xi_{j,t}^{\text{WD}} [(\mathbf{c}_t^j)^T \mathbf{z}_t - \mathbf{d}_t^j] \\ &+ \sum_{j \in \mathcal{E}^{\text{WD}}} \xi_{j,t-1}^{\text{WD}} [(\mathbf{c}_{t-1}^j)^T \mathbf{z}_{t-1} - \mathbf{d}_{t-1}^j] \\ &\leq \mathbf{d}_t^0 + \mathbf{d}_{t-1}^0 - (\mathbf{c}_t^0)^T \mathbf{z}_t - (\mathbf{c}_{t-1}^0)^T \mathbf{z}_{t-1}, t \in \mathcal{T} \end{aligned} \quad (31)$$

Since this is true for all cases,

$$\begin{aligned} &\max_{\mathbf{H}_t \boldsymbol{\xi}_t \leq \mathbf{h}_t} \left\{ \sum_{j \in \mathcal{E}^{\text{WD}}} \xi_{j,t}^{\text{WD}} [(\mathbf{c}_t^j)^T \mathbf{z}_t - \mathbf{d}_t^j] \right\} + \\ &\max_{\mathbf{H}_{t-1} \boldsymbol{\xi}_{t-1} \leq \mathbf{h}_{t-1}} \left\{ \sum_{j \in \mathcal{E}^{\text{WD}}} \xi_{j,t-1}^{\text{WD}} [(\mathbf{c}_{t-1}^j)^T \mathbf{z}_{t-1} - \mathbf{d}_{t-1}^j] \right\} \end{aligned}$$

$$\leq \mathbf{d}_t^0 + \mathbf{d}_{t-1}^0 - (\mathbf{c}_t^0)^T \mathbf{z}_t - (\mathbf{c}_{t-1}^0)^T \mathbf{z}_{t-1}, t \in \mathcal{T} \quad (32)$$

The final maximum in (32) is clearly:

$$\begin{aligned} &\sum_{j \in \mathcal{E}^{\text{WD}}} h_{j,t} |(\mathbf{c}_t^j)^T \mathbf{z}_t - \mathbf{d}_t^j| \\ &+ \sum_{j \in \mathcal{E}^{\text{WD}}} h_{j,t-1} |(\mathbf{c}_{t-1}^j)^T \mathbf{z}_{t-1} - \mathbf{d}_{t-1}^j|, \\ h_{j,t} &= \max\{|\bar{p}_{j,t}^{\text{WD}} - \hat{p}_{j,t}^{\text{WD}}|, |\hat{p}_{j,t}^{\text{WD}} - \underline{p}_{j,t}^{\text{WD}}|\}, \\ h_{j,t-1} &= \max\{|\bar{p}_{j,t-1}^{\text{WD}} - \hat{p}_{j,t-1}^{\text{WD}}|, |\hat{p}_{j,t-1}^{\text{WD}} - \underline{p}_{j,t-1}^{\text{WD}}|\} \end{aligned} \quad (33)$$

Thus, we can represent (33) as:

$$\begin{aligned} &(\mathbf{c}_t^0)^T \mathbf{z}_t + (\mathbf{c}_{t-1}^0)^T \mathbf{z}_{t-1} + \sum_{j \in \mathcal{E}^{\text{WD}}} h_{j,t} |(\mathbf{c}_t^j)^T \mathbf{z}_t - \mathbf{d}_t^j| \\ &+ \sum_{j \in \mathcal{E}^{\text{WD}}} h_{j,t-1} |(\mathbf{c}_{t-1}^j)^T \mathbf{z}_{t-1} - \mathbf{d}_{t-1}^j| \leq \mathbf{d}_t^0 + \mathbf{d}_{t-1}^0, t \in \mathcal{T} \end{aligned} \quad (34)$$

Then, (34) can be transformed into linear inequalities:

$$\Theta_{RC} = \left\{ (\mathbf{z}_t, \mathbf{z}_{t-1}) : \begin{array}{l} \exists \mathbf{u}_t \in \mathbb{R}^{|\mathcal{E}^{\text{WD}}|}, \mathbf{u}_{t-1} \in \mathbb{R}^{|\mathcal{E}^{\text{WD}}|}, \\ -\mathbf{u}_t^j \leq (\mathbf{c}_t^j)^T \mathbf{z}_t - \mathbf{d}_t^j \leq \mathbf{u}_t^j, \\ -\mathbf{u}_{t-1}^j \leq (\mathbf{c}_{t-1}^j)^T \mathbf{z}_{t-1} - \mathbf{d}_{t-1}^j \leq \mathbf{u}_{t-1}^j, \\ (\mathbf{c}_t^0)^T \mathbf{z}_t + (\mathbf{c}_{t-1}^0)^T \mathbf{z}_{t-1} \\ + \sum_{j \in \mathcal{E}^{\text{WD}}} h_{j,t} \mathbf{u}_t^j \\ + \sum_{j \in \mathcal{E}^{\text{WD}}} h_{j,t-1} \mathbf{u}_{t-1}^j \\ \leq \mathbf{d}_t^0 + \mathbf{d}_{t-1}^0, \\ j \in \mathcal{E}^{\text{WD}}, t \in \mathcal{T} \end{array} \right\} \quad (35)$$

where $\mathbf{u}_t$ and $\mathbf{u}_{t-1}$ denote auxiliary variables in robust constraints and $|\mathcal{E}^{\text{WD}}|$ denotes the number of wind farms in the EPS.

D. Compact Formulation

After the convex reformulation in Subsections A, B, and C, the original DRJCC model is transformed into a quadratic program with linear constraints. The compact formulation of the reformulation model is shown as follows:

$$\begin{aligned} \min_{\bar{\mathbf{x}}_E, \mathbf{x}_H^h, \mathbf{x}_B^h} f &= \bar{f}_E(\bar{\mathbf{x}}_E) + \sum_{h \in \mathcal{H}} f_H^h(\mathbf{x}_H^h, \mathbf{x}_B^h) \\ \text{s.t. } \bar{\mathbf{A}}_E \bar{\mathbf{x}}_E + \bar{\mathbf{A}}_{BE} \mathbf{x}_B^E &= \bar{\mathbf{a}}_E, \bar{\mathbf{B}}_E \bar{\mathbf{x}}_E + \bar{\mathbf{B}}_{BE} \mathbf{x}_B^E \leq \bar{\mathbf{b}}_H, \\ \mathbf{A}_H^h \mathbf{x}_H^h + \mathbf{A}_{BH}^h \mathbf{x}_B^h &= \mathbf{a}_H^h, \mathbf{B}_H^h \mathbf{x}_H^h + \mathbf{B}_{BH}^h \mathbf{x}_B^h \leq \mathbf{b}_H^h, \\ h &\in \mathcal{H} \end{aligned} \quad (36)$$

where we use the notation $\mathbf{x}_B^E$ to stack $\mathbf{x}_B^h$ over $h \in \mathcal{H}$; $\bar{f}(\cdot)$ denotes the objective function of the EPS after reformulation; $\bar{\mathbf{x}}_E$ denotes the vector of all decision variables of the EPS after reformulation; and $\bar{\mathbf{A}}_E$, $\bar{\mathbf{A}}_{BE}$, $\bar{\mathbf{a}}_E$, $\bar{\mathbf{B}}_E$, $\bar{\mathbf{B}}_{BE}$, and $\bar{\mathbf{b}}_H$ denote coefficient matrices of the linear constraints.

IV. DISTRIBUTED SOLUTION METHOD

In this section, a DBR algorithm is proposed for the reformulation model (36). The multiparametric programming and variable reduction methods are used together to formulate this dynamic boundary response efficiently. Notably, compared

with other multiparametric programming-based algorithms, the DBR algorithm here is computationally much cheaper because DHS operator does not need to explore all boundary response functions over the boundary space but dynamically obtains the desired boundary response function to avoid the “curse of dimensionality”. Moreover, the scale of DHS problem is further reduced by the variable reduction method.

A. DBR Algorithm

DBR algorithm is proposed to decompose the reformulation model (36) into an EPS subproblem and several DHS subproblems. The general idea of this algorithm is that the DHS operator creates the boundary response function and security constraints according to information from the EPS operator and then sends them to the EPS. The boundary response function represents the local marginal information of the optimal value function of the DHS under security constraints. The EPS operator adds the boundary response function and security constraints into the EPS subproblem to obtain the (local) optimum. In particular, we propose a variable reduction method to eliminate variables and constraints in the DHS to reduce the computation time of the boundary response function.

The detailed procedures of this DBR algorithm are shown as follows:

Step 1: Each DHS operator eliminates variables and constraints in DHS according to the variable reduction method.

Step 2: Each DHS operator sends boundary constraints $N_{BH}^h x_B^h \leq n_H^h$ to EPS operator.

Step 3: The iteration number k is initialized as 1. The EPS operator sends the feasible initial vector of the boundary variable $x_B^{(k)}$ to DHS operators.

Step 4: Dynamic Response: Each DHS operator takes $x_B^{(k)}$ as the parameters, obtains the *boundary response function* $\bar{f}_H^{h(k)}(x_B)$ and security constraints $\bar{S}^{h(k)} x_B + \bar{R}^{h(k)} \leq 0$ by projecting the internal variables onto the boundary based on multiparameter programming, and sends them to the EPS operator.

Step 5: EPS operator coordinates the boundary response function and security constraints from all DHS operators and optimizes (37) to obtain $x_B^{(k+1)}$:

$$\begin{aligned} \min_{x_E, x_B} & \bar{f}_E(\bar{x}_E) + \sum_{h \in \mathcal{H}} \bar{f}_H^{h(k)}(x_B^h) \\ \text{s.t.} & \bar{A}_E \bar{x}_E + \bar{A}_{BE} x_B^E = \bar{a}_E, \bar{B}_E \bar{x}_E + \bar{B}_{BE} x_B^E \leq \bar{b}_H, \\ & \bar{S}^{h(k)} x_B^h + \bar{R}^{h(k)} \leq 0, N_{BH}^h x_B^h \leq n_H^h, h \in \mathcal{H} \end{aligned} \quad (37)$$

Step 6: Convergence is checked. If $|x_B^{(k+1)} - x_B^{(k)}| \leq \delta$, where δ is a small positive threshold, the solution from the latest iteration is the optimal solution. Otherwise, the EPS operator sends the updated vector of the boundary variables $x_B^{(k+1)}$ to the DHS operators and then increases the iteration number k by 1. Return to *Step 4*.

In Subsections B and C, we describe the details of the DBR algorithm further.

B. Variable Reduction Method

The variable reduction method is employed in *Steps 1* and *2*, and it is a *preprocessing* step for the DHS operator before iteration. The variable reduction method aims to eliminate variables and constraints in DHS based on Gaussian elimination. Since variable reduction is a preprocessing step before iteration, which can be *done offline*, the matrix manipulation below will not cause additional computational overhead for the subsequent iterative process.

The motivation for this stems from the existence of a large number of equation constraints in the DHS. For example, for a 67-node 24-period DHS, there are 12,936 internal variables and 12,864 equation constraints. The equation constraints can be reduced by Gaussian elimination. The reduced system is equivalent to the original model in the subsequent boundary response.

In the rest of this section, we use an area of DHSs as an example, and the superscript h is omitted to simplify the notation.

DHS subproblem can be expressed as follows:

$$\begin{aligned} \min_{x_H, x_B} & f_H(x_H, x_B) \\ \text{s.t.} & A_H x_H + A_{BH} x_B = a_H, B_H x_H + B_{BH} x_B \leq b_H \\ & A_H \in \mathbf{R}^{m \times n}, A_{BH} \in \mathbf{R}^{m \times b}, a_H \in \mathbf{R}^{m \times 1} \\ & B_H \in \mathbf{R}^{r \times n}, B_{BH} \in \mathbf{R}^{r \times b}, b_H \in \mathbf{R}^{r \times 1} \end{aligned} \quad (38)$$

Suppose x_H is a feasible solution of (38); divide x_H , A_H , and B_H into two submatrices:

$$\begin{aligned} x_H &= \begin{bmatrix} x_{HI} \\ x_{HN} \end{bmatrix}, x_{HI} \in \mathbf{R}^{m \times 1}, x_{HN} \in \mathbf{R}^{(n-m) \times 1} \\ A_H &= [A_{HI} \ A_{HN}], B_H = [B_{HI} \ B_{HN}] \\ A_{HI} &\in \mathbf{R}^{m \times m}, A_{HN} \in \mathbf{R}^{m \times (n-m)} \\ B_{HI} &\in \mathbf{R}^{r \times m}, B_{HN} \in \mathbf{R}^{r \times (n-m)} \end{aligned} \quad (39)$$

The criterion for dividing x_H is that A_H should be a nonsingular square matrix. In this DHS model, we set the heating production of heating boilers as x_{HI} and the other variables in DHS as x_{HN} .

Thus, the constraints of (38) can be expressed as:

$$A_{HI} x_{HI} + A_{HN} x_{HN} + A_{BH} x_B = a_H \quad (41)$$

$$B_{HI} x_{HI} + B_{HN} x_{HN} + B_{BH} x_B \leq b_H \quad (42)$$

Then, x_{HN} can be solved by (41):

$$x_{HN} = A_{HN}^{-1} (a_H - A_{HI} x_{HI} - A_{BH} x_B) \quad (43)$$

Substituting (43) into (42), we have:

$$(B_{HI} - B_{HN} A_{HN}^{-1} A_{HI}) x_{HI} + (B_{BH} - B_{HN} A_{HN}^{-1} A_{BH}) x_B \leq b_H - B_{HN} A_{HN}^{-1} a_H \quad (44)$$

Define $E_H = B_{HI} - B_{HN} A_{HN}^{-1} A_{HI}$, $E_{BH} = B_{BH} - B_{HN} A_{HN}^{-1} A_{BH}$, and $e_H = b_H - B_{HN} A_{HN}^{-1} a_H$. Then, (44) can be rewritten as:

$$E_H x_{HI} + E_{BH} x_B \leq e_H \quad (45)$$

Moreover, the redundant inequalities in (45) can be removed by the redundancy checking method in [39].

The coefficient matrix of objective function (14) can also be divided similarly to (39):

$$\begin{aligned} f_H(\mathbf{x}_{HI}, \mathbf{x}_{HN}, \mathbf{x}_B) &= [\mathbf{x}_{HI}^T \quad \mathbf{x}_{HN}^T] \begin{bmatrix} \mathbf{Q}_{HI} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_{HN} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{HI} \\ \mathbf{x}_{HN} \end{bmatrix} \\ &+ [\mathbf{x}_{HI}^T \quad \mathbf{x}_{HN}^T] \begin{bmatrix} \mathbf{Q}_{HBI} \\ \mathbf{Q}_{HBN} \end{bmatrix} \mathbf{x}_B + [\mathbf{D}_{HI} \quad \mathbf{D}_{HN}] \begin{bmatrix} \mathbf{x}_{HI} \\ \mathbf{x}_{HN} \end{bmatrix} \\ &+ \mathbf{x}_B^T \mathbf{Q}_{BB} \mathbf{x}_B + \mathbf{D}_B \mathbf{x}_B + e_h \end{aligned} \quad (46)$$

Then, (46) can be replaced by:

$$\begin{aligned} f_H^{VR}(\mathbf{x}_{HI}, \mathbf{x}_B) &= \mathbf{x}_{HI}^T \mathbf{Q}_{HI}^{VR} \mathbf{x}_{HI} + \mathbf{x}_{HI}^T \mathbf{Q}_{HIB}^{VR} \mathbf{x}_B \\ &+ \mathbf{x}_B^T \mathbf{Q}_{BB}^{VR} \mathbf{x}_B + \mathbf{D}_{HI}^{VR} \mathbf{x}_{HI} \\ &+ \mathbf{D}_{HB}^{VR} \mathbf{x}_B + e_H^{VR} \end{aligned} \quad (47)$$

where

$$\begin{aligned} \mathbf{Q}_{HI}^{VR} &= \mathbf{Q}_{HI} + (\mathbf{A}_{HN}^{-1} \mathbf{A}_{HI})^T \mathbf{Q}_{HN} \mathbf{A}_{HN}^{-1} \mathbf{A}_{HI} \\ \mathbf{Q}_{HIB}^{VR} &= \mathbf{Q}_{HBI} + 2(\mathbf{A}_{HN}^{-1} \mathbf{A}_{HI})^T \mathbf{Q}_{HN} \mathbf{A}_{HN}^{-1} \mathbf{A}_{BH} \\ &\quad - (\mathbf{A}_{HN}^{-1} \mathbf{A}_{HI})^T \mathbf{Q}_{HBN} \\ \mathbf{Q}_{BB}^{VR} &= \mathbf{Q}_{BB} + (\mathbf{A}_{HN}^{-1} \mathbf{A}_{BH})^T \mathbf{Q}_{HN} \mathbf{A}_{HN}^{-1} \mathbf{A}_{BH} \\ &\quad + (\mathbf{A}_{HN}^{-1} \mathbf{A}_{HI})^T \mathbf{Q}_{HBN} \\ \mathbf{D}_{HI}^{VR} &= \mathbf{D}_{HI} - \mathbf{D}_{HN} \mathbf{A}_{HN}^{-1} \mathbf{A}_{HI} \\ &\quad - 2(\mathbf{A}_{HN}^{-1} \mathbf{a}_H)^T \mathbf{Q}_{HN} \mathbf{A}_{HN}^{-1} \mathbf{A}_{HI} \\ \mathbf{D}_{HB}^{VR} &= \mathbf{D}_{HB} - \mathbf{D}_{HN} \mathbf{A}_{HN}^{-1} \mathbf{A}_{BH} + (\mathbf{A}_{HN}^{-1} \mathbf{a}_H)^T \mathbf{Q}_{HBN} \\ &\quad - 2(\mathbf{A}_{HN}^{-1} \mathbf{a}_H)^T \mathbf{Q}_{HN} \mathbf{A}_{HN}^{-1} \mathbf{A}_{BH} \\ e_H^{VR} &= (\mathbf{A}_{HN}^{-1} \mathbf{a}_H)^T \mathbf{Q}_{HN} \mathbf{A}_{HN}^{-1} \mathbf{a}_H + \mathbf{D}_{HN} \mathbf{A}_{HN}^{-1} \mathbf{a}_H + e_h \end{aligned}$$

For the subsequent discussion, we extract the constraints for $\mathbf{x}_B$ alone in (45) and divide (45) into $\mathbf{M}_H \mathbf{x}_H + \mathbf{M}_{BH} \mathbf{x}_B \leq \mathbf{m}_H$ and $\mathbf{N}_{pH} \mathbf{x}_R \leq \mathbf{n}_H$. Thus, DHS subproblem (38) is replaced by:

$$\begin{aligned} \min_{\mathbf{x}_{HI}, \mathbf{x}_B} \quad & f_H^{VR}(\mathbf{x}_{HI}, \mathbf{x}_B) \\ \text{s.t.} \quad & \mathbf{M}_H \mathbf{x}_{HI} + \mathbf{M}_{BH} \mathbf{x}_B \leq \mathbf{m}_H, \mathbf{N}_{BH} \mathbf{x}_B \leq \mathbf{n}_H \end{aligned} \quad (48)$$

C. Boundary Response Function

The boundary response function is calculated in *Step 4*. The theoretical basis of this algorithm is multiparameter quadratic programming [39], where the optimal objective function can be expressed as a piecewise quadratic function of parameters. However, to avoid the “curse of dimensionality” in traditional multiparameter programming, this paper, inspired by the critical region projection (PCR) in [13], does not need to construct the entire piecewise quadratic function but iteratively obtains the desired boundary response function and security constraints.

Boundary vector $\mathbf{x}_B$ is taken as a parameter in the DHS subproblem (48), and the KKT condition of (48) is:

$$\begin{bmatrix} 2\mathbf{Q}_{HI}^{VR} & \{\mathbf{M}_H^T\}_A \\ \{\mathbf{M}_H\}_A & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x}_{HI}^* \\ \{\boldsymbol{\mu}\}_A \end{bmatrix} = \begin{bmatrix} -\mathbf{D}_{HI}^{VR^T} - \mathbf{Q}_{HIB}^{VR} \mathbf{x}_B \\ \{\mathbf{m}_H - \mathbf{M}_{BH} \mathbf{x}_B\}_A \end{bmatrix} \quad (49)$$

$$\{\mathbf{M}_H \mathbf{x}_{HI}^* + \mathbf{M}_{BH} \mathbf{x}_B - \mathbf{m}_H\}_A = 0, \{\boldsymbol{\mu}\}_A > 0 \quad (50)$$

$$\{\mathbf{M}_H \mathbf{x}_{HI}^* + \mathbf{M}_{BH} \mathbf{x}_B - \mathbf{m}_H\}_I < 0, \{\boldsymbol{\mu}\}_A = 0 \quad (51)$$

where $\{\cdot\}_A$ and $\{\cdot\}_I$ denote active constraints and inactive constraints, respectively.

After the preprocessing step in Subsection B, the size of the coefficient matrix in (49) is significantly reduced. If the coefficient matrix is nonsingular, (49) can be solved:

$$\begin{bmatrix} \mathbf{x}_{HI}^* \\ \{\boldsymbol{\mu}\}_A \end{bmatrix} = \begin{bmatrix} \mathbf{T}_{11} & \mathbf{T}_{12} \\ \mathbf{T}_{21} & \mathbf{T}_{22} \end{bmatrix} \begin{bmatrix} -\mathbf{D}_{HI}^{VRT} - \mathbf{Q}_{HB}^{VR} \mathbf{x}_B \\ \{\mathbf{m}_H - \mathbf{M}_{BH} \mathbf{x}_B\}_A \end{bmatrix} \quad (52)$$

If the coefficient matrix is singular, the problem degenerates. To eliminate this problem, we need to find a maximal linearly independent group $\{\mathbf{M}_H\}_A$, and the detailed method is shown in [14].

Then, $\mathbf{x}_{HI}^*$ and $\{\boldsymbol{\mu}\}_A$ are affine functions of $\mathbf{x}_B$:

$$\begin{aligned} \mathbf{x}_{HI}^* &= \mathbf{L}_H \mathbf{x}_B + \mathbf{K}_H \\ \mathbf{L}_H &= -\mathbf{T}_{11} \mathbf{Q}_{HB}^{VR} - \mathbf{T}_{12} \{\mathbf{M}_{BH}\}_A \end{aligned} \quad (53)$$

$$\mathbf{K}_H = -\mathbf{T}_{11} \mathbf{D}_{HI}^{VRT} + \mathbf{T}_{12} \{\mathbf{m}_H\}_A$$

$$\{\boldsymbol{\mu}\}_A = \mathbf{L}_A \mathbf{x}_B + \mathbf{K}_A \geq 0$$

$$\mathbf{L}_A = -\mathbf{T}_{21} \mathbf{Q}_{HIB}^{VR} - \mathbf{T}_{22} \{\mathbf{M}_{BH}\}_A \quad (54)$$

$$\mathbf{K}_A = -\mathbf{T}_{21} \mathbf{D}_{HI}^{VRT} + \mathbf{T}_{22} \{\mathbf{m}_H\}_A$$

The inactive constraints can also be expressed by $\mathbf{x}_B$:

$$\begin{aligned} \{\mathbf{M}_H \mathbf{x}_{HI}^* + \mathbf{M}_{BH} \mathbf{x}_B - \mathbf{m}_H\}_I &= \mathbf{L}_I \mathbf{x}_B + \mathbf{R}_I \leq 0 \\ \mathbf{L}_I &= \{\mathbf{M}_H\}_I (-\mathbf{T}_{11} \mathbf{Q}_{HB}^{VR} - \mathbf{T}_{12} \{\mathbf{M}_{BH}\}_A) + \{\mathbf{M}_{BH}\}_I \\ \mathbf{K}_I &= \{\mathbf{M}_H\}_I (-\mathbf{T}_{11} \mathbf{D}_{HI}^{VRT} + \mathbf{T}_{12} \{\mathbf{m}_H\}_A) - \{\mathbf{m}_H\}_I \end{aligned} \quad (55)$$

Substitute (53) into objective function (47) to obtain the boundary response function:

$$\bar{f}_H^{(k)}(\mathbf{x}_B) = \mathbf{x}_B^T \bar{\mathbf{Q}}_B^{(k)} \mathbf{x}_B + \bar{\mathbf{D}}_B^{(k)} \mathbf{x}_B + \bar{e}_B^{(k)} \quad (56)$$

where

$$\begin{aligned} \bar{\mathbf{Q}}_B^{(k)} &= \mathbf{L}_H^T \mathbf{Q}_{HI}^{VR} \mathbf{L}_H + \mathbf{L}_H^T \mathbf{Q}_{HIB}^{VR} + \mathbf{Q}_{BB}^{VR} \\ \bar{\mathbf{D}}_B^{(k)} &= \mathbf{K}_H^T \mathbf{L}_H + 2\mathbf{K}_H^T \mathbf{Q}_{HI}^{VR} \mathbf{L}_H + \mathbf{D}_{HI}^{VR} \mathbf{L}_H + \mathbf{D}_{HB}^{VR} \\ \bar{e}_B^{(k)} &= \mathbf{K}_H^T \mathbf{Q}_{HI}^{VR} \mathbf{K}_H + \mathbf{D}_{HI}^{VR} \mathbf{K}_H + e_H^{VR} \end{aligned}$$

Notably, the boundary response function (56) is a “local optimum”. The optimum condition of (56) is that KKT condition (43) does not change, namely, (54) and (55) are satisfied. Thus, (54) and (55) together constitute the security constraints:

$$\bar{\mathbf{S}}^{(k)} \mathbf{x}_B + \bar{\mathbf{R}}^{(k)} \leq 0, \bar{\mathbf{S}}^{(k)} = \begin{bmatrix} -\mathbf{L}_A \\ \mathbf{L}_I \end{bmatrix}, \bar{\mathbf{R}}^{(k)} = \begin{bmatrix} -\mathbf{K}_A \\ \mathbf{K}_I \end{bmatrix} \quad (57)$$

The DHS operator only needs to transfer the boundary response function (56) and security constraints (57) to the EPS operator.

Remark 1: In model (48), the constraint $\mathbf{N}_{BH} \mathbf{x}_B \leq \mathbf{n}_H$ is not used in creating the boundary response function. However, ignoring this constraint may make subproblems of the DHS infeasible. To address this problem, the constraint $\mathbf{N}_{BH} \mathbf{x}_B \leq \mathbf{n}_H$ also needs to be sent to the EPS operator in advance (namely, in *Step 2*) so the EPS operator can use it.

V. CASE STUDIES

A. Case Settings

A 24-period dispatch problem of an IEHS with a 24-node EPS and two 67-node DHSs is demonstrated in this paper to verify the advantages of the proposed DRJCC model and distributed algorithm. The EPS consists of two wind farms and six thermal units. The 67-node DHS consists of three CHP units, which come from the first 67 nodes of the city-level testbed in [40]. The CHP units in the two DHSs are connected to buses 1, 2, and 16 and buses 21, 22, and 23 of the EPS. Two wind farms are connected to buses 6 and 8. The topology of the IEHS test system is shown in Fig. 2.

The construction procedure of the ambiguity set refers to that in [36], and wind output data come from southeastern Australia between 2012 and 2013 [41]. The risk parameters ε_1 and ε_2 are set to 5%. These problems were solved by GUROBI in MATLAB R2020a on a computer with an Intel Core i7-11700 CPU and 16 GB of memory.

In what follows, we first explain the necessity of using mixed distributionally robust joint chance constraints and robust constraints in the model; then, we showcase the effectiveness of our DBR-based decentralized method.

B. Out-of-Sample Performance of the DRJCC Model

In this subsection, a sample set with 50 samples from the data source is used to obtain the optimal value, and an out-of-sample set with 2000 samples is used to test the out-of-sample performance. Two performance metrics, namely, the out-of-sample cost $\hat{F}$ and out-of-sample empirical joint violation probability (EJVP) $\hat{Y}^j$, are assessed to quantitatively evaluate the out-of-sample performance.

$$\hat{F} = \sum_{t \in \mathcal{T}} \left\{ c(\tilde{y}_t) + \frac{1}{M} \sum_{i=1}^M c_0^T \hat{\alpha} \mathbf{1}^T \hat{\xi}_{N+i,t} \right\} \quad (58)$$

$$\hat{Y}^j = \frac{1}{|\mathcal{T}|} \sum_{t \in \mathcal{T}} \left\{ \frac{1}{M} \sum_{i=1}^M \mathbb{I}_{A^j(\tilde{Y}_t) \hat{\xi}_{N+i,t} > b^j(\tilde{y}_t)} \right\} \quad (59)$$

where $\tilde{y}_t$ and $\tilde{Y}_t$ denote the optimal values of y_t and Y_t , respectively; $c(\tilde{y}_t)$ denotes the deterministic term in (6); $|\mathcal{T}|$ denotes the number of dispatch periods; and M denotes the number of test samples.

The out-of-sample cost and EJVP (average of 24 periods) of the joint chance constraints and individual chance constraints are shown in Fig. 3. As seen in Fig. 3, the out-of-sample cost increases with increasing Wasserstein radius ρ , while EJVP gradually decreases, which is in line with expectations. This is because a larger Wasserstein radius leads to a more conservative solution, i.e., a higher cost and lower violation probabilities. Additionally, compared with the joint chance constraints, the EJVP of the individual chance constraints decreases more slowly with increasing Wasserstein radius ρ and is higher than the required risk parameters, so the individual chance constraints cannot probabilistically ensure security constraint satisfaction.

Meanwhile, we explore the impacts of the joint and individual chance constraints on the resulting operational decisions. The total downward and upward reserve capacities are shown in Fig. 4. As shown in Fig. 4, both the total downward and upward reserve capacities increase with increasing Wasserstein radius ρ . Compared with individual chance constraints, the joint chance constraint can set aside more reserve capacity to address the uncertainty of wind power and ensure the security of the system. In addition, hourly electric power dispatch with $\rho = 10^{-4}$ is provided in Fig. 5.

In joint chance constraints, the risk parameter ε is an important parameter that reflects the system operator's risk attitude. We explore the impacts of different risk parameters on the resulting out-of-sample cost with $\rho = 10^{-4}$, and the

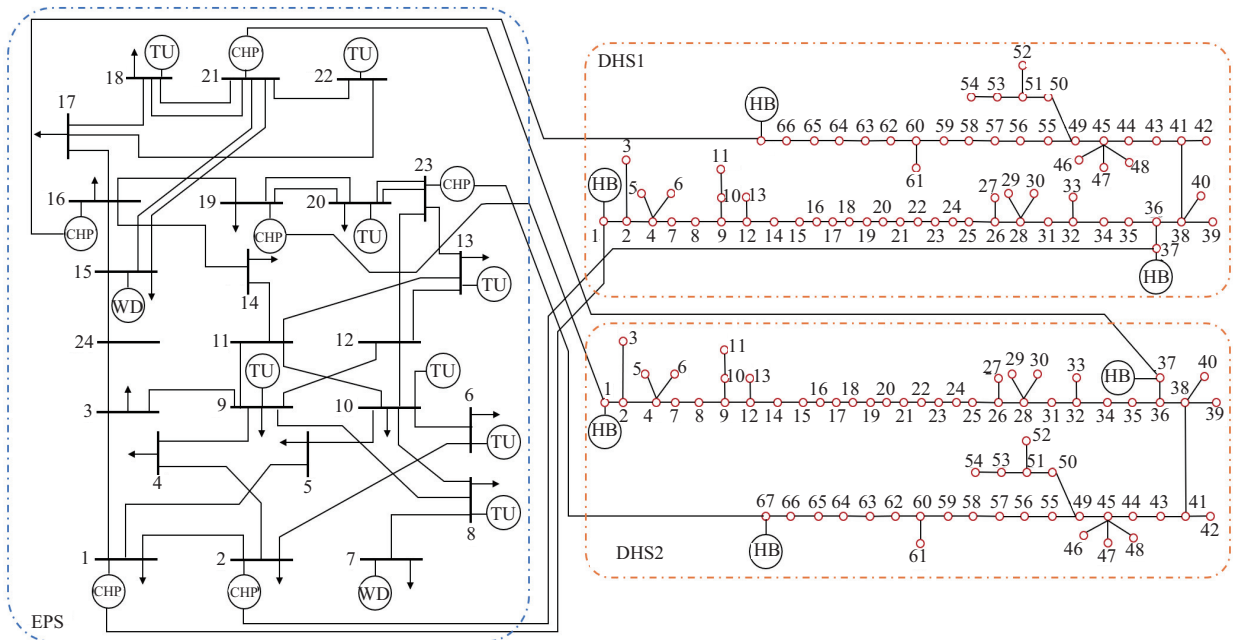


Fig. 2. Topology of the IEHS test system with a 5-bus power system and two 67-node district heating systems.

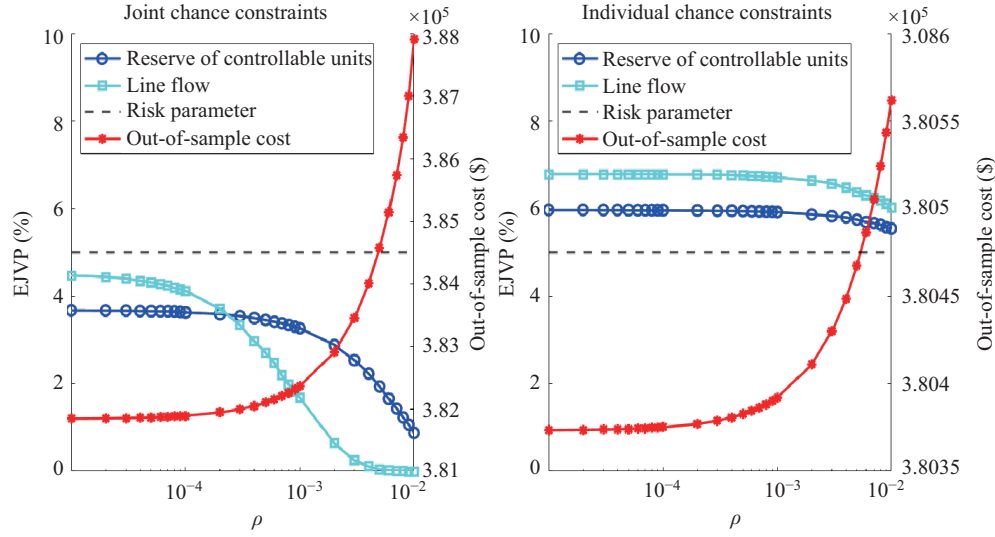


Fig. 3. Out-of-sample cost and EJVP of joint chance constraints and individual chance constraints under different radii.

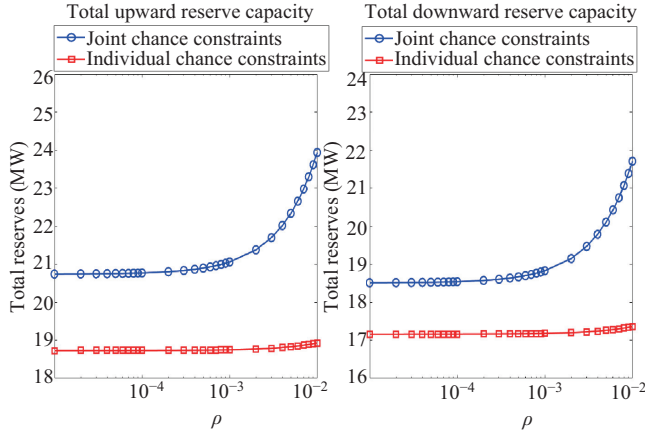


Fig. 4. Total downward and upward reserve capacity of joint chance constraints and individual chance constraints under different radii.

results are shown in Table I. As seen from Table I, the out-of-sample cost decreases as the risk factor increases, which means that operators are exposed to a greater joint violation probability.

TABLE I
PERFORMANCE COMPARISON

ε (%)	Cost (\$)
2	381,967.4
3	381,923.6
5	381,888.5
10	381,862.2
20	381,849.1

Moreover, we compare using chance constraints and robust constraints for the ramping constraints with $\rho = 10^{-4}$ in Table II. Compared to chance constraints, the cost of robust constraints is higher, but robust constraints can guarantee that the ramping constraint offers 100% reliability against uncertainty, which is in line with expectations. Therefore, for non-violable physical constraints, such as ramping constraints, it is necessary to use robust constraints.

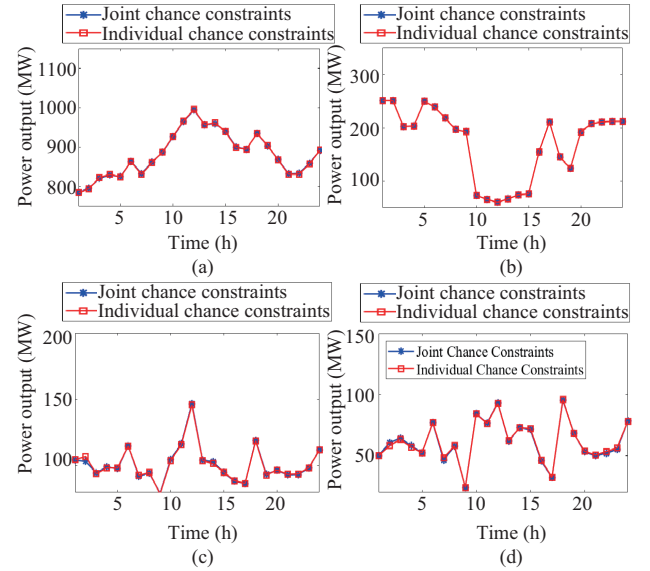


Fig. 5. Hourly electric power dispatch of (a) total thermal units, (b) total wind farms, (c) total CHP units in DHS1, and (d) total CHP units in DHS2 with $\rho = 10^{-4}$.

TABLE II
PERFORMANCE COMPARISON

Method	Chance constraints	Robust constraints
EJVP (%)	2.59	0
Cost (\$)	375,866.1	381,888.5

C. Performance of the DBR Algorithm

The performance of DBR algorithm is discussed below. First, the iteration number and CPU time of DBR algorithm are compared with those of the traditional centralized method, classical ADMM algorithm [42], adaptive ADMM [43], and HGD algorithm [10] for the DRJCC model of an IEHS with $\rho = 10^{-4}$ and $\delta = 10^{-6}$. The results are shown in Table III.

It can be seen in Table III that both DBR algorithm and ADMM algorithm can obtain the same optimal solution

TABLE III
PERFORMANCE COMPARISON

Method	Iteration number	CPU time (s)	Overall cost (\$)	Gap (%)
Centralized	—	15.20	381,888.5	—
DBR	2	27.97	381,888.5	0
Classical ADMM	91	1386.66	381,888.5	0
Adaptive ADMM	49	653.31	381,888.5	0
HGD	>500	—	—	—

as the centralized method. However, HGD algorithm cannot obtain the solution for this problem due to local convergence. Compared with the classical ADMM and adaptive ADMM algorithms, the iteration number of DBR algorithm proposed in this paper is reduced by 24 times and 45 times, respectively, which has obvious advantages.

To verify the effectiveness of the preprocessing step, we compared the computation time of the DBR algorithm and the PCR-based algorithm [13] on the DHS subproblem. The results are shown in Table IV. Compared with the PCR algorithm, DBR is 30 times faster due to the preprocessing step for DHSs, thus reducing the overall time during the iterative process. In addition, the preprocessing step of each DHS is approximately 32.6 seconds, which is perfectly acceptable for an offline process.

TABLE IV
PERFORMANCE COMPARISON

Method	Iteration number	DHS subproblem time (s)	Overall cost (\$)
DBR	2	3.28	373,433.0
PCR-based	2	105.72	373,433.0

VI. CONCLUSION

We consider joint chance constraints and propose a DRJCC multiperiod dispatch model for an IEHS. Moreover, robust constraints are employed in the physical constraints to fully protect against uncertainty. To handle mixed distributionally robust chance constraints and conventional robust constraints, we present a method that combines Bonferroni and CVaR approximations to transform the original model into a quadratic program. Additionally, a DBR algorithm is proposed for the reformulation model, which includes a preprocessing step for the DHS to resolve the computational inefficiency issue faced by many other multiparametric programming-based algorithms.

Case studies show the proposed DRJCC model can guarantee the constraints are smaller than the risk parameters and that simultaneously, the mixed robust constraints can be obeyed 100% against uncertainty for non-violable physical constraints. Additionally, the DBR-based decentralized algorithm has obvious advantages over the standard decentralized algorithms, and the preprocessing step can greatly reduce the amount of time in the interactive process.

Future research will investigate the optimized approximation for decentralized distributionally robust joint chance constraints to reduce conservatism.

APPENDIX

For completeness, the detailed DHS model is provided as

TABLE AI
NOTATIONS IN DHSS

Sets	
$\mathcal{H}^{\text{HB}}$	Index set of heating boilers in the DHS
$\mathcal{H}^{\text{nd}}$	Index set of nodes in the DHS
$\mathcal{H}^{\text{pipe}}$	Index set of pipelines in DHS
$\mathcal{H}^{\text{LD}}$	Index set of heating loads in the DHS
$S_n^{\text{pipe-}}/S_n^{\text{pipe+}}$	Index set of pipelines ending/starting at node n
$\text{CHP}_s^{\text{HS}}/\text{HB}_s^{\text{HS}}$	Index set of CHP units/heating boiler connected to heating station s
$\text{Nd}_s^{\text{HS}}/\text{Nd}_l^{\text{LD}}$	Index set of nodes connected to heating station s /heating load l
Parameters	
$a_{1,i}/a_{2,i}/a_{4,i}/a_{5,i}$	Cost coefficients of CHP unit i
r_h^{HB}	Cost coefficient of heating boiler h
P_i^k/H_i^k	Electricity/heating production corresponding to the k th extreme point in the feasible operation region of CHP unit i
$\bar{h}_{h,t}^{\text{HB}}$	Maximum heating production of heating boiler h in period t
$\bar{p}_i^{\text{CHP}}/\underline{p}_i^{\text{CHP}}$	Maximum/minimum power production of CHP h in period t
η_h^{HB}	The conversion efficiency of heating boiler h in period t
λ_b	Cross-sectional area of pipeline b
L_b	Mass flow rate of water in pipeline b in supply/return networks
ρ_w	Density of water
c	Specific heating capacity of water
A_b	Cross sectional area of pipeline b
L_b	Length of pipeline $ba_{0,i}$
$d_{l,t}$	Demand of heat load l in period t
$\tau_n^{\text{NS}}/\bar{\tau}_n^{\text{NS}}$	Lower/upper temperature limits of supply network node n
$\tau_n^{\text{NR}}/\bar{\tau}_n^{\text{NR}}$	Lower/upper temperature limits of return network node n
$C_{i,t}^{\text{CHP}}(\cdot)$	Cost function of CHP unit i in period t
$C_{h,t}^{\text{HB}}(\cdot)$	Cost function of heating boiler h in period t
Decision variables	
$h_{i,t}^{\text{HB}}/f_{i,t}^{\text{HB}}$	Heating production/fuel input of heating boiler i in period t
$\tau_{n,t}^{\text{NS}}/\tau_{n,t}^{\text{NR}}$	Temperature at node n of supply and return network in period t
$\bar{\tau}_{b,t}^{\text{PS,out}}/\bar{\tau}_{b,t}^{\text{PR,out}}$	Temperature at the outlet of pipeline b of supply/return network in period t when ignoring the heating loss of mass flow.
$\tau_{b,t}^{\text{PS,out}}/\tau_{b,t}^{\text{PR,out}}$	Temperature at the outlet of pipeline b of supply/return network in period t
$\tau_{b,t}^{\text{PS,in}}/\tau_{b,t}^{\text{PR,in}}$	Temperature at the inlet of pipeline b of supply/return network in period t

follows along with an explanation of notations in Table AI. More detail can be found in [10].

1) Objective Function:

CHP Unit Cost:

$$C_{i,t}^{\text{CHP}}(p_{i,t}^{\text{CHP}}, h_{i,t}^{\text{CHP}}) = a_{0,i} + a_{1,i}p_{i,t}^{\text{CHP}} + a_{2,i}h_{i,t}^{\text{CHP}} + a_{3,i}(p_{i,t}^{\text{CHP}})^2 + a_{4,i}(h_{i,t}^{\text{CHP}})^2 + a_{5,i}p_{i,t}^{\text{CHP}}h_{i,t}^{\text{CHP}}, \quad i \in \mathcal{H}^{\text{CHP}}, t \in \mathcal{T} \quad (\text{A1})$$

Heating Boiler Cost:

$$C_{h,t}^{\text{HB}}(f_{h,t}^{\text{HB}}) = r_h^{\text{HB}} f_{h,t}^{\text{HB}}, \quad h \in \mathcal{H}^{\text{HB}}, t \in \mathcal{T} \quad (\text{A2})$$

2) Constraints:

CHP Unit Operation Constraints:

$$p_{i,t}^{\text{CHP}} = \sum_{k=1}^{NK_i} \alpha_{i,t}^k P_{i,t}^k, h_{i,t}^{\text{CHP}} = \sum_{k=1}^{NK_i} \alpha_{i,t}^k H_{i,t}^k, i \in \mathcal{H}^{\text{CHP}}, t \in \mathcal{T} \quad (\text{A3})$$

$$\sum_{k=1}^{NK_i} \alpha_{i,t}^k = 1, i \in \mathcal{H}^{\text{CHP}}, t \in \mathcal{T} \quad (\text{A4})$$

$$0 \leq \alpha_{i,t}^k \leq 1, i \in \mathcal{H}^{\text{CHP}}, t \in \mathcal{T} \quad (\text{A5})$$

CHP Unit Output Constraints:

$$\underline{p}_i^{\text{CHP}} \leq p_{i,t}^{\text{CHP}} \leq \bar{p}_i^{\text{CHP}}, i \in \mathcal{H}^{\text{CHP}}, t \in \mathcal{T} \quad (\text{A6})$$

Heating Boiler Operation Constraints:

$$h_{h,t}^{\text{HB}} = \eta_h^{\text{HB}} f_{h,t}^{\text{HB}}, h \in \mathcal{H}^{\text{HB}}, t \in \mathcal{T} \quad (\text{A7})$$

Heating Boiler Operation Constraints:

$$0 \leq h_{h,t}^{\text{HB}} \leq \bar{h}_{h,t}^{\text{HB}}, h \in \mathcal{H}^{\text{HB}}, t \in \mathcal{T} \quad (\text{A8})$$

Heating Source Constraints:

$$h_{h,t}^{\text{HB}} + h_{i,t}^{\text{CHP}} = c \cdot m_n^{\text{DHS}} \cdot (\tau_{n,t}^{\text{NS}} - \tau_{n,t}^{\text{NR}}) \quad (\text{A9})$$

$$h \in \text{HB}_s^{\text{HS}}, i \in \text{CHP}_s^{\text{HS}}, n \in N_d^{\text{HS}}, t \in \mathcal{T}$$

Temperature Mixing Constraints:

$$\sum_{b- \in S_n^{\text{pipe}}} (\tau_{b-,t}^{\text{PS,out}} \cdot ms_{b-}^{\text{pipe}}) = \tau_{n,t}^{\text{NS}} \sum_{b+ \in S_n^{\text{pipe}+}} ms_{b+,t}^{\text{pipe}} \quad (\text{A10})$$

$$n \in \mathcal{H}^{\text{nd}}, t \in \mathcal{T}$$

$$\sum_{b- \in S_n^{\text{pipe}-}} (\tau_{b-,t}^{\text{PR,out}} \cdot mr_{b-}^{\text{pipe}}) = \tau_{n,t}^{\text{NR}} \sum_{b+ \in S_n^{\text{pipe}+}} mr_{b+,t}^{\text{pipe}} \quad (\text{A11})$$

$$n \in \mathcal{H}^{\text{nd}}, t \in \mathcal{T}$$

Heating Loss Constraints:

$$\begin{aligned} \bar{\tau}_{b,t}^{\text{PS,out}} &= (1 - \lceil \phi_b^{\text{S}} \rceil + \phi_b^{\text{S}}) \tau_{b,t- \lceil \phi_b^{\text{S}} \rceil}^{\text{PS,in}} \\ &+ (\lceil \phi_b^{\text{S}} \rceil - \phi_b^{\text{S}}) \tau_{b,t- \lceil \phi_b^{\text{S}} \rceil + 1}^{\text{PS,in}}, b \in \mathcal{H}^{\text{pipe}}, t \in \mathcal{T} \end{aligned} \quad (\text{A12})$$

$$\begin{aligned} \bar{\tau}_{b,t}^{\text{PR,out}} &= (1 - \lceil \phi_b^{\text{R}} \rceil + \phi_b^{\text{R}}) \tau_{b,t- \lceil \phi_b^{\text{R}} \rceil}^{\text{PR,in}} \\ &+ (\lceil \phi_b^{\text{R}} \rceil - \phi_b^{\text{R}}) \tau_{b,t- \lceil \phi_b^{\text{R}} \rceil + 1}^{\text{PR,in}}, b \in \mathcal{H}^{\text{pipe}}, t \in \mathcal{T} \end{aligned} \quad (\text{A13})$$

$$\phi_b^{\text{S}} = \frac{\rho_w A_b L_b}{ms_b^{\text{pipe}}}, \phi_b^{\text{R}} = \frac{\rho_w A_b L_b}{mr_b^{\text{pipe}}}, b \in \mathcal{H}^{\text{pipe}} \quad (\text{A14})$$

Temperature Quasi-Dynamics Constraints:

$$\begin{aligned} \tau_{b,t}^{\text{PS,out}} &= \tau_t^{\text{am}} + (\bar{\tau}_{b,t}^{\text{PS,out}} - \tau_t^{\text{am}}) \\ &\times \exp \left[-\frac{\lambda_b \Delta t}{A_b \rho c} (\lceil \phi_b^{\text{S}} \rceil - 0.5) \right], b \in \mathcal{H}^{\text{pipe}}, t \in \mathcal{T} \end{aligned} \quad (\text{A15})$$

$$\begin{aligned} \tau_{b,t}^{\text{PR,out}} &= \tau_t^{\text{am}} + (\bar{\tau}_{b,t}^{\text{PR,out}} - \tau_t^{\text{am}}) \\ &\times \exp \left[-\frac{\lambda_b \Delta t}{A_b \rho c} (\lceil \phi_b^{\text{R}} \rceil - 0.5) \right], b \in \mathcal{H}^{\text{pipe}}, t \in \mathcal{T} \end{aligned} \quad (\text{A16})$$

Inlet Temperature Constraints:

$$\tau_{b+,t}^{\text{PS,in}} = \tau_{n,t}^{\text{NS}}, b+ \in S_n^{\text{pipe}+}, t \in \mathcal{T} \quad (\text{A17})$$

$$\tau_{b-,t}^{\text{PR,in}} = \tau_{n,t}^{\text{NR}}, b- \in S_n^{\text{pipe}-}, t \in \mathcal{T} \quad (\text{A18})$$

Temperature Security Constraints:

$$\underline{\tau}_n^{\text{NS}} \leq \tau_{n,t}^{\text{NS}} \leq \bar{\tau}_n^{\text{NS}}, n \in \mathcal{H}^{\text{nd}}, t \in \mathcal{T} \quad (\text{A19})$$

$$\underline{\tau}_n^{\text{NR}} \leq \tau_{n,t}^{\text{NR}} \leq \bar{\tau}_n^{\text{NR}}, n \in \mathcal{H}^{\text{nd}}, t \in \mathcal{T} \quad (\text{A20})$$

Heating Load Constraints

$$c \cdot m_n^{\text{DHS}} \cdot (\tau_{n,t}^{\text{NS}} - \tau_{n,t}^{\text{NR}}) = d_{l,t},$$

$$n \in N_d^{\text{LD}}, l \in \mathcal{H}^{\text{LD}}, t \in \mathcal{T} \quad (\text{A21})$$

In the compact form of the DHS model, the objective function (14) includes (A1) and (A2); equality constraints (15) include (A3), (A4), (A7), (A9), (A10), (A11)–(A13), (A15)–(A18), and (A21); inequality constraints (16) include (A5), (A6), (A8), (A19), and (A20).

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